

COMMENTS ON “ARITHMETIC ELLIPTIC CURVES
IN GENERAL POSITION”

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(1.) The notation “ $\text{ord}_v(-) : F_v \rightarrow \mathbb{Z}$ ” in the final sentence of the first paragraph following Definition 1.1, should read “ $\text{ord}_v(-) : F_v^\times \rightarrow \mathbb{Z}$ ”.

(2.) In Definition 1.2, (ii), the *non-resp’d* and *first resp’d* items in the display should be *reversed!* That is to say, the notation “ $\alpha \lesssim_{\mathcal{F}} \beta$ ” corresponds to “ $\alpha(x) - \beta(x) \leq C$ ”; the notation “ $\alpha \gtrsim_{\mathcal{F}} \beta$ ” corresponds to “ $\beta(x) - \alpha(x) \leq C$ ”.

(3.) The first portion of the first sentence of the statement of Corollary 4.4, should read: “Let $\bar{\mathbb{Q}}$ be an algebraic closure of \mathbb{Q} ; ...”.

(4.) The “ $\text{log-diff}_{\overline{\mathcal{M}_{\text{ell}}}}([E_L])$ ” in the second inequality of the final display of the statement of Corollary 4.4, should read “ $\text{log-diff}_{\overline{\mathcal{M}_{\text{ell}}}}([E_L])$ ”.

(5.) The *equality*

$$\text{ht}_E \approx (\deg(E)/\deg(\omega_X)) \cdot \text{ht}_{\omega_X}$$

implicit in the final “ \approx ” of the final display of the proof of Theorem 2.1, should be replaced by an *inequality*

$$\text{ht}_E \lesssim 2 \cdot (\deg(E)/\deg(\omega_X)) \cdot \text{ht}_{\omega_X}$$

[which follows immediately from Proposition 1.4, (ii)], and the expression “ $\deg(E)/\deg(\omega_X)$ ” in the inequality imposed on the *choice* of ϵ' should be replaced by the expression “ $2 \cdot (\deg(E)/\deg(\omega_X))$ ”.

(6.) The phrase “Corollary 2.1” in the first paragraph of §2, should read “Theorem 2.1”.

(7.) The inequality “ $x_{\mathcal{S}_\circ} \leq x_{\mathcal{S}_\bullet}$ ” at the end of the second to last sentence of the proof of Corollary 4.3, should read “ $x_{\mathcal{S}_\circ} \leq x_{\mathcal{S}_\bullet}$ ”.

(8.) Suppose that we are in the situation of Example 1.3, (ii). Let $U \subseteq X$ be an open subscheme. Then a “*compactly bounded subset*”

$$\mathcal{K}_V \subseteq U(\bar{\mathbb{Q}}) \quad (\subseteq X(\bar{\mathbb{Q}}))$$

of $U(\overline{\mathbb{Q}})$ is to be understood as a subset which forms a compactly bounded subset of $X(\overline{\mathbb{Q}})$ [i.e., in the sense discussed in Example 1.3, (ii)] and, moreover, satisfies the property that for each $v \in V^{\text{arc}} \stackrel{\text{def}}{=} V \cap \mathbb{V}(\mathbb{Q})^{\text{arc}}$ (respectively, $v \in V^{\text{non}} \stackrel{\text{def}}{=} V \cap \mathbb{V}(\mathbb{Q})^{\text{non}}$), the compact domain $\mathcal{K}_v \subseteq X^{\text{arc}}$ (respectively, $\mathcal{K}_v \subseteq X(\overline{\mathbb{Q}}_v)$) is, in fact, *contained in*

$$U(\mathbb{C}) \subseteq X(\mathbb{C}) = X^{\text{arc}} \quad (\text{respectively, } U(\overline{\mathbb{Q}}_v) \subseteq X(\overline{\mathbb{Q}}_v)).$$

In particular, this convention should be applied to the use of the term “compactly bounded subset” in the statements of Theorem 2.1, Lemma 3.7, Theorem 3.8, Corollary 4.4. Although this convention was not discussed explicitly in Example 1.3, (ii), it is, in effect, discussed *explicitly* in the discussion of “compactly bounded subsets” at the beginning of the Introduction. Moreover, this convention is *implicit* in the arguments involving compactly bounded subsets in the proof of Theorem 2.1.

(9.) In the discussion following the second display of Example 1.3, (ii), the phrase “(respectively, $X(\mathbb{Q}_v)$)” should read “(respectively, $X(\overline{\mathbb{Q}}_v)$)”.

(10.) The first display of the paragraph immediately following Remark 3.3.1, should read as follows:

$$|\alpha|^2 \stackrel{\text{def}}{=} \left| \int_{E_v} \alpha \wedge \bar{\alpha} \right|$$

[i.e., the integral should be replaced by the absolute value of the integral].

(11.) In the discussion following the final display of Example 1.3, (ii), the phrase “to a subset” should read “to a subset of”.

(12.) In the text immediately following the first display of Definition 1.5, (iv), the phrase “*conductor of x* ” should read “*conductor of x [with respect to D]*”.

(13.) The following phrase should be added to the end of the third sentence [i.e., “Here, the condition of ...”] of §3:

[cf. the conclusion concerning the condition “(b)” in Lemma 3.7].

(14.) In the discussion in the paragraph immediately preceding Lemma 3.2 of the *extension class* associated to the exact sequence of G_K -modules of the second display of this paragraph, it should be recalled that there is a *natural isomorphism* between the group of extensions $\text{Ext}_{G_K}^1(\mathbb{F}_l, \mathbb{F}_l(1))$ and the cohomology module $H^1(G_K, \mathbb{F}_l(1))$.

(15.) In the proof of Theorem 3.8, the text “[which has the effect of ...” should read “[so that choosing a level structure on the $3 \cdot 5$ -torsion points has the effect of ...”.

(16.) In the proof of Theorem 3.8, the text “finite primes of L ” should read “finite primes of L' ”.